Gravitational backreaction of kinky cosmic strings

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- Kinks are generated by the reconnection process. The lifetime of kinks due to gravitational radiation is an interesting question.
- Our computation gives finite results of the backreaction. We find a class of kinks string which does not smooth out apparently. (comparing to the lifetime of the whole loop)

Nambu-Goto Action

$$S = -\mu \int d\sigma d\tau (-h)^{1/2}$$

light-cone coordinates

$$u = \tau + \sigma$$
$$v = \tau - \sigma$$

τ, σ, u, v are dimensionless parameters. We choose the gauge,

$$g_{\alpha\beta}\partial_u x^\alpha \partial_u x^\beta = 0$$
$$g_{\alpha\beta}\partial_v x^\alpha \partial_v x^\beta = 0$$

The classical equation of motion is

$$\partial_u \partial_v x^\mu = -\Gamma^\mu_{\alpha\beta}\partial_u x^\alpha \partial_v x^\beta$$

String loop in flat spacetime

The equation is simplified,

$$\partial_u \partial_v x^\mu = 0$$

Then we put further gauge conditions,

$$x^0 = l\tau = \frac{l}{2}(u+v)$$

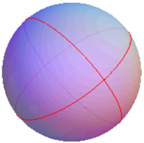
If u, v are set to have period 1, the factor l is the invariant length of the string loop. The period of the string loop is $l/2$. We have

$$|\partial_u \mathbf{x}(u)| = |\partial_v \mathbf{x}(v)| = l/2$$

This leads to the "unit" sphere representation.

Unit sphere representation, cusp

$\partial_u \mathbf{x}(u)$ and $\partial_v \mathbf{x}(v)$ are two curves on a sphere with radius $l/2$.



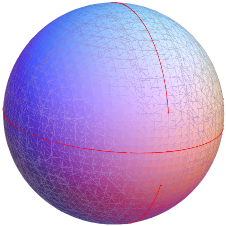
When $\partial_u \mathbf{x}(u)$ and $\partial_v \mathbf{x}(v)$ intersects, we get a cusp:

$$\partial_u \mathbf{x}(u_0) = \partial_v \mathbf{x}(v_0)$$

which implies the cusp point $x^\mu(u_0, v_0)$ reaches the velocity of light.

A cusp on the string locally looks like the plane curve $x^2 = y^3$. It appear only once in one period and emit a strong GW beam which is in perpendicular to the cusp plane.

Unit sphere representation, kink



When $\partial_u \mathbf{x}(u)$ or $\partial_v \mathbf{x}(v)$ is not continuous, we get a kink. The larger the separation, the sharper the angle on the string loop.

A kink is moving along the string loop with the velocity of light. It also emit strong GW, but, which is not a single beam.

Cusp vs. Kink

If both $\partial_u \mathbf{x}(u)$ or $\partial_v \mathbf{x}(v)$ are smooth, generically, they will intersect and result in cusps on the string loop. However, the discontinuous points on the two branches can avoid the intersection and reduce the number of cusps.

- Cusps $\rightarrow$ Kinks. Self-intersecting string loops can break into smaller "daughter loops" by reconnection. Each daughter loop get a pair of kinks and the original cusps may disappear.
- Kinks $\rightarrow$ Cusps? If the kinks are smoothed out by the gravitational radiation quickly (comparing to the lifetime of the whole loop), the separations become smaller and finally we may get new cusps.

The lifetime of the whole loop is given by the radiation formula.

Radiation formula

Damour and Vilenkin, gr-qc/0104026.
The fourier transform of the energy-momentum tensor is

$$T^{\mu\nu}(k) = \frac{4\mu}{l} \int_0^1 du \int_0^1 dv \; \partial_u x^{(\mu} \partial_v x^{\nu)} e^{-i(k_m \cdot X)}$$

where

$$k = (\omega, \mathbf{k}) = \frac{4\pi}{l} m(1, \mathbf{n}).$$

Let $X^\mu(u,v) = X_+^\mu(u) + X_-^\mu(v)$, then we can separate $T^{\mu\nu}$,

$$T^{\mu\nu}(k) = \frac{4\mu}{l} I_+^{(\mu} I_-^{\nu)}, \;\; I_+^\mu = \int_0^1 du \; \partial_u x^\mu e^{-i(k_m \cdot X_+)}$$

The far field is given by,

$$\bar{h}_{\mu\nu}(t, \mathbf{x}) = \frac{4G \sum_\omega e^{-i\omega(t-r)} T_{\mu\nu}(\omega, \mathbf{k})}{r} + O(r^{-2})$$

Radiation power

Weinberg, gravitation and cosmology

$$\frac{dP_m}{d\Omega} = \frac{G\omega_m^2}{\pi}[T^{*\mu\nu}(\omega_m, \mathbf{k})T_{\mu\nu}(\omega_m, \mathbf{k}) - \frac{1}{2}|T_\mu^\mu(\omega_m, \mathbf{k})|^2]$$

- Cusps. $P_m \propto m^{-4/3}$
- Kinks. $P_m \propto m^{-s}, s > 4/3$

Total radiation power

$$P \sim \Gamma G\mu^2, \ \Gamma \sim 40 \text{ to } 60$$

The lifetime of the string loop,

$$t_{loop} \sim \frac{\mu l}{P} \sim \frac{l}{\Gamma G\mu}$$

The momentum, angular momentum radiation can also be derived from $T^{\mu\nu}$.

Gravitational backreaction of the cosmic string

It is not easy to study the string loop deform directly from the radiation computation. So we need the backreaction method which deal with the "near field". The method is given by

Quashnock and Spergel, Physics Rev D42, 8.

The gravitational backreaction problems is *divergence-free*. Q&S argued this by the equivalence principle and also verified it by analytical formula.

Backreaction formula

In weak field approximation, the equation of motion is,

$$\partial_u \partial_v x_\gamma = -\frac{1}{2}(\partial_\alpha h_{\beta\gamma} + \partial_\beta h_{\alpha\gamma} - \partial_\gamma h_{\alpha\beta})\partial_u x^\alpha \partial_v x^\beta$$

The derivative of the gravitational field is given by the retard Green function,

$$\partial_\gamma h_{\mu\nu}(x) = 8G\mu \int dudv\ F_{\mu\nu}(z(u,v)) \cdot \partial_\gamma[\theta(x^0 - z^0)\delta((x-z)^2)]$$

where

$$F_{\mu\nu}(x(u,v)) = \partial_u x_\mu \partial_v x_\nu + \partial_v x_\mu \partial_u x_\nu - \eta_{\mu\nu}\partial_u x^\alpha \partial_v x_\alpha$$

It is very hard to solve them simultaneously, so we

- use the free string loop to get $\partial_\gamma h_{\mu\nu}(x)$.
- put $\partial_\gamma h_{\mu\nu}(x)$ in the equation of motion to get $\partial_u \partial_v x_\gamma$
- integrate $\partial_u \partial_v x_\gamma$ to acquire the perturbed string loop

Backreaction formula

$$F_{\mu\nu}(z(u_0, v_0))\partial_u x^\mu = 0, etc.$$

The derivative on the theta will give a delta function $\delta^4(x - z)$, which vanishes when it is combined with $\partial_u x^\mu$. So the formula is simplified as

$$\partial_u \partial_v x_\gamma(u_0, v_0) = 16G\mu \int dudv \;\; Y_\gamma(u, v)\theta(x^0 - z^0)\delta'(f)$$

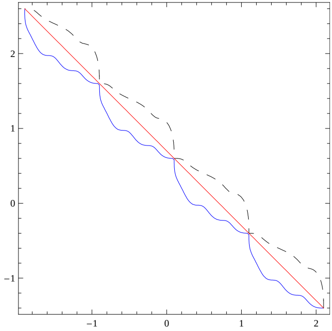
where $f(u, v) = (z(u, v) - z(u_0, v_0))^2$ and,

$$\begin{aligned} Y_\gamma(u, v) &= -\frac{1}{2}\Big[F_{\beta\gamma}(z(u, v))(x_\alpha - z_\alpha) + F_{\alpha\gamma}(z(u, v))(x_\beta - z_\beta) \\ &- F_{\alpha\beta}(z(u, v))(x_\gamma - z_\gamma)\Big] \cdot \partial_u x^\alpha \partial_v x^\beta \end{aligned}$$

Only the zero-locus of f have contribution to this integral.

Null Curve

A typical diagram of $f(u,v)=0$ is shown on the left diagram.



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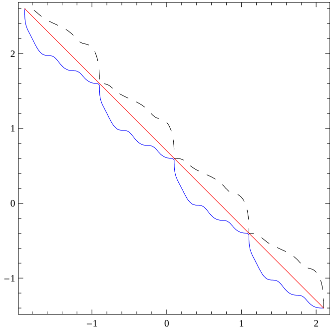
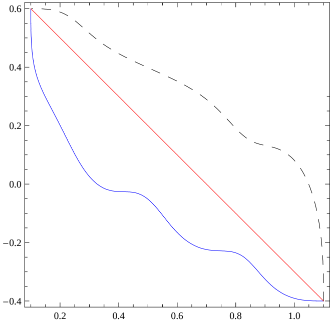
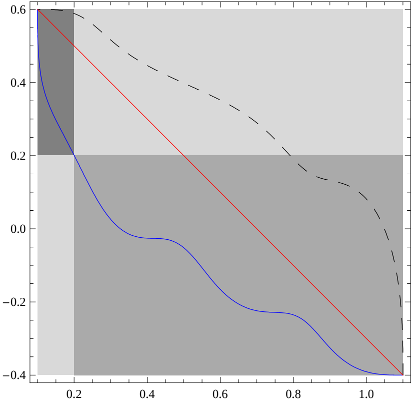




Because of the period of $\sigma = (u - v)/2$, the null curve contains identical sub-diagrams. We just need to choose one representative. (right diagram)

2d integral → 1d integral

The null curve determines the implicit function $v_{ret}(u)$ or $u_{ret}(v)$. We need to separate the retarded curve into $u - branch$ and $v - branch$.



2d integral → 1d integral

Now we can reduce the 2d integral with integrating by parts, for example, in the $u - branch$

$$\partial_u \partial_v x_\gamma(u_0, v_0) \supset 16G\mu \int_{us}^{u_0+1} du \left[\frac{Y_\gamma(u,v)}{\partial_v f} \delta(f) \right]_{v=vs}^{v=v_0-1} - 16G\mu \int_{us}^{u_0+1} \frac{du}{\partial_v f} \partial_v \left[\frac{Y_\gamma(u,v)}{\partial_v f} \right]_{v=v_{ret}(u)} \quad (1)$$

The second integral is finite since Y vanishes to high orders near the field point. The first integral is trivial and will turn to two boundary terms. One at (us, vs) is finite. The other one at $(u_0 + 1, v_0 - 1)$ is exactly on the field point which should vanish by $Y(u_0, v_0)$ is zero.

Change of the string loop

Given $\partial_u \partial_v x_\gamma(u_0, v_0)$, we can find the average change in one period

$$\Delta(\partial_u x^\mu)(u) = \int_0^1 dv\ \partial_u \partial_v x^\mu(u, v).$$

The gauge condition $\partial_u \tilde{x}^\alpha \partial_u \tilde{x}_\alpha = \partial_v \tilde{x}^\alpha \partial_v \tilde{x}_\alpha = 0$ still holds, However, $\tilde{x}^0$ is not proportional to τ now. We need to reparameterize,

$$\tilde{u} = u + h(u) = u - \frac{\Delta l}{l} u + \frac{2}{l} \int_0^u ds\ \Delta(\partial_u x^0)(s)$$

$\partial_{\tilde{u}} \tilde{x}^\alpha$ will satisfy all the gauge conditions used. We will compare $\partial_{\tilde{u}} \tilde{x}^\alpha$ and $\partial_u x^\alpha$.

Change of the string loop

Q&S proved a useful relation

$$\Delta P^{\mu} = 2\mu \int dudv \partial_u \partial_v x^{\mu}$$

It is a easy way to compute the total energy and momentum radiation in one period. (even more convenient than Damour and Vilenkin). It is consistent with our Δl.

Purpose

Q&S used the analytical method above to study cusp case. However, they used an approximate way for the kinky case. (They used 32 segments to represent a string loop with 8(16) kinks.) Their conclusion is that kink is smoothed out much quicker than the string loop.

$$t_{kink} \sim \frac{l_{kink}}{\Gamma G\mu}$$

It is also difficult to define the kink size l_{kink}.
We want to study a string loop without cusps but with a few kinks by the analytical way. We are interested in whether the new cusps will form and also the deform of the loop.

Program

Our program is powered by Mathematica 6. For a smooth string loop, it cost less than one second to get a $\partial_u \partial_v x^\mu(u_0, v_0)$. The analytical backreaction method also works for the kink case because the singularity induced by the kink point is integrable. For a kinky string loop, it cost about three seconds. We compute $\partial_u \partial_v x^\mu(u_0, v_0)$ on a 50 by 50 lattice in the 1 by 1 worldsheet. When the computation is finished, we can get the new string loop and also the energy momentum radiation.

Check

Although we are not going to work on the cusp backreaction, the cusp case is a good check of our program. We computed Burden curve and verified the following,

- the result is independent of the choice of (us,vs)
- the result is consistent with the program used the original 2d integral, by David
- the result gives the consistent result of energy radiation (error, $< 2\%$)

Simplest kinky model

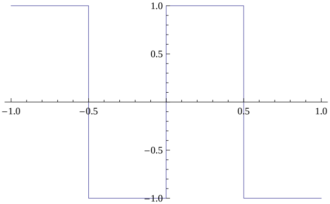
We find the "simplest kinky model",

$$\partial_u x(u) = l/2(u, cos(2\pi u), sin(2\pi u), 0)$$

and

$$\partial_v x(v) = l/2(v, 0, 0, f(v))$$

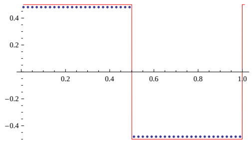
where $f(v)$ is



This model contains two kinks which are right angles.

The backreaction result of this model is summarized as

- There is no change of the components which are zero originally.
- The new $\partial_u x^1(u)$ and $\partial_u x^2(u)$ are still sine functions with decreased amplitude and phase shift
- f(v) is changed to



Therefore this model decays in a self-similar way, i.e., the shape is invariant by the gravitional backreaction. Its radiation power is about $38G\mu^2$. (I guess that it has the weakest radiation power in all string loops without kinks on both two component)

We need to consider more realistic kinky loops. We find a class of string loop which is parameterized by Δ. u-component is still a circle but the v-component is made of two segments of one circle. The angle between the two lines which connect each ending point and the sphere center is $2\pi\Delta$.

Our backreaction results are

- $\Delta = 1/10$. In one period the angle is changed from 36 degrees to 34.5 degrees, i.e. 4% decrease. The radiation power is $46G\mu^2$ which implies that the energy loss in one period is about 12%.
- $\Delta = 1/36$. In one period the angle is changed from 10 degrees to 9.96 degrees, i.e. almost invariant. The radiation power is $50G\mu^2$ which implies that the energy loss in one period is about 12.7%.

In either case, the decay of the kink is slower than the shrink of the loop.

Further cases

- We can make the kinks more "local"